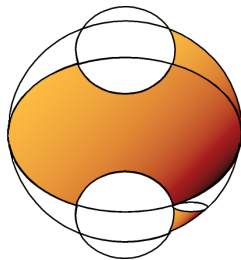


Primary Numeracy

A mapping, review and analysis of
Australian research in numeracy learning
at the primary school level

Compendium



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The report and the CD ROM will be available from the Clearinghouse for National Literacy and Numeracy Research. Orders can be placed via the website: www.gu.edu.au/school/cls/clearinghouse

Printed by JS McMillan Printing Group

ISBN: 0 642 77435 8

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The views expressed herein do not necessarily represent the views of the Australian Government Department of Education, Science and Training.

This project was funded under the Australian Government's Numeracy Research and Development Initiative.

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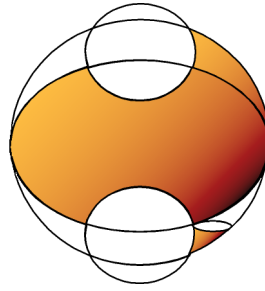
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The results of this project are detailed in a full written report. This compendium is only a summary, so details of projects, smaller studies, and some areas of research identified have not been included. The full report provides further research summaries, project details, and references. The CD that accompanies this compendium includes the full report as well as an electronic database of about 200 projects and over 700 publications.

Timeframe of the data

The projects and publications included in the database and report are indicative of the range of mathematics education research and development reports from the period from 1990 to 2003.

Primary Numeracy Concept Map



<p>Assessment</p>	<ul style="list-style-type: none"> Achievement Assessment techniques Assessment programs Diagnostic assessment School entry assessment <ul style="list-style-type: none"> International assessment National assessment Statewide assessment
<p>Broader contexts</p>	<ul style="list-style-type: none"> Literature reviews Major reports System initiatives
<p>Classroom practice</p>	<ul style="list-style-type: none"> Grouping Intervention Pedagogy Resources Teaching strategies <ul style="list-style-type: none"> Teaching aids Technology Textbooks Motivating students Problem solving and investigations Questioning and discussion Real world contexts <ul style="list-style-type: none"> Calculators Computers
<p>Students</p>	<ul style="list-style-type: none"> Gifted Informal learning Learning styles Student attitudes Students at risk
<p>Teachers</p>	<ul style="list-style-type: none"> Pre-service Professional development Teacher beliefs Teacher change Teacher effects Teacher knowledge

<p>Curriculum and processes</p>	<p>Concept development</p> <p>Curriculum issues</p> <p>Developmental frameworks</p> <p>Mathematical thinking</p> <p>Using mathematics</p>	<p>Algebra</p> <p>Chance & data</p> <p>Measurement</p> <p>Number</p> <p>Space</p> <p>Children's problem solving</p> <p>Children's thinking and strategies</p> <p>Language of mathematics</p> <p>Visualisation</p> <p>Computation</p> <p>Counting</p> <p>Decimals</p> <p>Estimation</p> <p>Fractions</p> <p>Number sense</p> <p>Percentages</p> <p>Place value and the Number System</p>
<p>Equity</p>	<p>Disability</p> <p>Ethnicity</p> <p>Gender</p> <p>Indigenous</p> <p>Language factors</p> <p>LBOTE</p> <p>Rural</p> <p>Socio-economic status</p>	
<p>School community</p>	<p>Community</p> <p>Parents</p> <p>Primary-secondary transition</p> <p>School factors</p>	

Figure 1: Themes and sub-themes used to organise the Primary Numeracy database.

Primary Numeracy: A Mapping, Review and Analysis of Australian Research in Numeracy Learning at the Primary School Level

A major policy objective of the Australian Government is to provide all young people in Australia with strong foundations in numeracy and English literacy skills.

This project was funded by the Australian Government Department of Education, Science and Training, under the *National Strand of the Numeracy Research and Development Initiative*, to provide a mapping of Australian research on numeracy learning at the primary school level during the last decade, to assist in program and policy development.

The information outlined was gathered from government and non-government agencies, universities, and mathematics education and other organisations, with the assistance of Australian and international consultants. A web-based instrument was also used to gather data directly from members of the Australian mathematical education community. The data were organised into the themes and sub-themes that form the basis for Figure 1.

This compendium is a brief summary of the full report. It provides an overview of the findings of the Australian research into primary numeracy, identifies perceived gaps in the research, and suggests directions for future research.

Included with this printed compendium is a CD containing the full report, together with an indexed electronic database with details of approximately 200 projects and over 700 publications. This material is also available from the Australian Government Department of Education, Science and Training's website.

The International Context

The increased emphasis on numeracy development that was evident in Australian political and educational circles during the latter half of the 1990s was part of a movement that was apparent in many countries. The driving force behind these initiatives around the world was the expectation that all children should become numerate citizens.

In the United Kingdom, the *National Numeracy Project* proposed that classes should provide opportunities for pupils to talk; be listened to; receive feedback; explain their knowledge, thinking and methods; and suggest alternative ways of tackling problems. The resulting *National Numeracy Strategy*, first implemented in 1999, emphasised mental calculation; 45 to 60 minutes of mathematics daily; a template for lessons; the use of a week-by-week framework; and a national professional development program.

In the USA, the National Council of Teachers of Mathematics *Standards* for curriculum, assessment, and professional development took a socially situated view of learning, and stressed the need for children to develop their own mathematical meanings.

In New Zealand, the key curriculum document *Mathematics in the New Zealand Curriculum* outlined not only expected curriculum content, but also ways of developing essential skills, including: communication, numeracy, information, problem solving, self-management and competitiveness, as well as social, co-operative, physical, work and study skills.

There has been considerable international research into what constitutes effective teaching of numeracy, with a wide range of factors found to be associated with high student achievement. These factors include:

- high teacher expectations of students;
- a focus on children's mathematical learning, rather than on the provision of pleasant classroom experiences;
- providing a challenging curriculum;
- the use of higher-order questioning;
- making connections both within mathematics and between mathematics and different contexts; and
- the use of highly interactive teaching that involves students in class discussion.

Nevertheless, there are also many conflicting findings, together with considerable difficulty in characterising effective teachers and less effective teachers. This has important implications for policy and research regarding the improvement of mathematics achievement in schools.

The Australian Context

In 1999, all Education Ministers endorsed new national goals for schooling in the twenty-first century (MCEETYA, 1999) — known as the *Adelaide Declaration*. The numeracy and English literacy goal agreed by Ministers is that students should have “attained the skills of numeracy and English literacy; such that, every student should be numerate, able to read, write, spell and communicate at an appropriate level” (MCEETYA, 1999, p. 4). The National Literacy and Numeracy Plan, agreed by the Australian Government Minister and all State and Territory Ministers in 1997, provides a coherent framework for working towards the achievement of the national literacy and numeracy goal, through a coordinated approach by the Australian, State and Territory Governments to improving students' literacy and numeracy outcomes.

The National Literacy and Numeracy Plan sets expectations for numeracy and literacy achievement in Australian schools. It involves the early identification of students at risk and the use of associated intervention strategies, the development of literacy and numeracy benchmarks, and professional development for teachers. Disadvantaged groups have been targeted, with funding made available for initiatives, including the *Strategic Assistance for Improving Student Outcomes Programme*, *The National Indigenous English Literacy and Numeracy Strategy*;

Literacy and Numeracy Development in the Middle Years of Schooling (Beyond the Middle); and the *Quality Teacher Programme*.

A major program to support the Australian Government's National Literacy and Numeracy Plan is the *Numeracy Research and Development Initiative*, comprising four national projects and ten projects undertaken by State and Territory government and non-government school authorities.

The remainder of this compendium discusses the Australian research reviewed, the gaps identified and possible directions for future Australian research.

Equity and the School Community

In keeping with the Australian Government's policy to provide all young people in Australia with strong foundations in numeracy and English literacy, disadvantaged groups with lower numeracy and English literacy achievement — including Indigenous Australians — are regarded as a target for special attention. In terms of the Australian research into primary numeracy reviewed here, considerably more research focused on individual and social-group differences between children than on broader community socio-cultural factors that may affect children's learning of mathematics — for example, home environments, parental attitudes and community programs.

Equity

Among the various themes grouped under *Equity*, research into the learning needs and mathematical achievements of Indigenous students was the most prolific. Research into gender issues was also well represented. There was noticeably less research, however, relating to *disability*, *ethnicity*, and *rural* issues, despite the fact that large proportions of the Australian population fall under these umbrellas.

Disability and learning difficulties

The review of the research clearly supports the findings of the *Students with Disabilities Project* (van Kraayenoord, Elkins, Palmer, & Rickards, 2000) that there is a lack of published literature in the area of numeracy development for students with all types of disabilities. There has been greater emphasis on literacy than on numeracy when dealing with students with learning difficulties resulting from physical, social and psychological factors. Teacher aids, many with little numeracy training, take on much of the instruction in classrooms.

In many Australian states, there are statewide, as well as localised, programs aimed at identifying children "at risk" of experiencing numeracy difficulties — particularly those with learning difficulties. Appropriate teacher professional development was clearly identified as a critical element contributing to the success of these programs, with teachers achieving success by using a more problem-solving, activity-based approach together with a wider range of assessment procedures. Inclusive programming that provided detailed and visible accountability was found to provide a powerful planning tool for teachers.

Based on the research reviewed, it is evident that there has been little specific research aimed at:

- identifying specific groups of children encountering learning difficulties in numeracy;
- the provision of quality numeracy teaching for target groups in the early years, including intervention programs where needed; and
- extending the current focus on the prevention or remediation of numeracy difficulties in the early years to target groups in the later years of schooling.

Very little research attention has been directed at the numeracy difficulties and needs of children with physical disabilities. However, the Australian Government has committed funding of \$4.5 million under the *National Literacy and Numeracy Strategies and Projects Programme* to assist in equipping teachers to better meet the needs of students with disabilities and learning difficulties. Findings from the resulting projects are likely to help fill this gap.

Ethnicity and English as a second language

Research findings on the numeracy achievements of children from language backgrounds other than English (LBOTE) appear inconsistent. Although some research attention has been directed to the numeracy learning of bilingual children, little is known about the effects of bilingualism on either the learning outcomes for these children, or on how teachers in mainstream, multicultural classes deal with cultural diversity.

Comprehension problems cause errors in solving mathematics word problems, with error frequency related to competency in both the mother tongue and the language of instruction. Data from the *Third International Mathematics and Science Study* (TIMSS) revealed that the lowest achievers have families who use a language other than English at home while the high achievers are often children born in non-English speaking countries whose families have adopted English as their home language.

Socio-economic status and rural students

Research on children attending schools in disadvantaged areas has revealed a greater range in numeracy achievement than for children from schools in other areas. The research indicates that successful numeracy learning for children in disadvantaged areas occurs when:

- families and community members are involved in numeracy programs;
- teachers' expectations of children are high;
- high expectations are conveyed clearly to parents and other significant community members;
- teaching is shaped by the results of teacher-child interviews; and
- open-ended learning tasks are used.

Only one study of rural children who were not Indigenous was identified — a study of on-line learning of numeracy in rural and urban settings.

Gender

Issues associated with the attainment of gender equity were found to be the most researched of all equity dimensions, apart from Indigenous issues. There has been, however, a marked decrease in the extent of this kind of research since the 1980s, with no major projects from the last decade with gender as the main focus. The review of the Australian research found:

- gender differences in overall numeracy achievement to be virtually non-existent, although there were small (non-significant) differences by content area that are consistent with those of the past;
- boys to be usually more confident than girls about their mathematical prowess; and
- gender-stereotyped attitudes and perceptions of girls' and boys' capabilities persisting, although to a lesser extent than in the past.

There appears to be little research evidence at the primary level — in Australia and elsewhere — to support the perceptions and claims that boys, rather than girls, are now the disadvantaged group with respect to numeracy learning.

Indigenous students

Data on the numeracy achievements of Australia's Indigenous students consistently reveal that the numeracy levels of Indigenous students are generally lower than those of other Australians. Findings from several small, action-research based studies have revealed a range of factors that may be implicated in this persistent pattern of achievement. These factors include:

- different learning needs of Indigenous children;
- the language of instruction being different from home community languages;
- incompatibility between classroom practices and children's background experience;
- low teacher expectations; and
- strained teacher-student relationships.

Success in addressing Indigenous students' needs has been achieved through:

- appropriate professional development activities;
- drawing on and integrating community knowledge and Indigenous culture into the numeracy curriculum;
- promoting positive self-identity; and
- exploiting children's preferred learning styles.

While there has been considerable research relating to Indigenous students, much of the research into factors that may enhance or impede achievement has been action research involving small numbers of students. There is a need for nationwide, co-ordinated research studies, involving members of Indigenous communities, that expand understandings of the needs of Indigenous learners and assist in the development of appropriate numeracy curricula.

The Australian Government's current *National Indigenous English Literacy and Numeracy Strategy*, which involves all Australian States and Territories in supporting the identification and dissemination of effective practice models and teaching methods drawn from earlier pilot projects, will partially fill this gap.

The school community

Evidence supports the view that all children will have optimum opportunities when strong educational partnerships between school, community, and home are developed. In Victoria, the *Early Literacy Research Project* (Hill & Crévola, 1998) developed a model comprising nine interrelated elements designed to improve

learning outcomes in literacy, numeracy, and other curriculum areas. These design elements included affective, home, and school-related factors (e.g., leadership, teaching programs, school and class organisation, and special needs). The Hill-Crévola model has been used as the basis of several numeracy projects in Victoria, including the *Early Numeracy Research Project*.

In summary, while there has been a significant amount of research in the areas of *Equity* and the *School community*, particularly in the learning needs and mathematical achievements of Indigenous students and issues relating to gender, there has been considerably less research into issues relating to disability, ethnicity, socio-economic and rural factors, and into school organisation. This is particularly true for research into the needs of students with disabilities, where much of the research focuses on literacy rather than numeracy.

In view of the National Literacy and Numeracy Plan's strong focus on ensuring that all students, including the most educationally disadvantaged, attain sound literacy and numeracy skills, there is still a need for large-scale research into the most effective ways to meet Indigenous students' numeracy achievement needs.

Teachers, Students and Classroom Practice

Over half the total research reviewed focused on issues associated with *Teachers*, *Students* and *Classroom practice*, with most of this research being carried out in classroom settings.

Teachers

A significant amount of Australian numeracy research in the past decade has focused on teachers, their training and professional development, teacher change, teacher's beliefs, and, to a lesser extent, their content and pedagogical content knowledge.

Pre-service teacher education

Findings from a number of studies relating to pre-service teachers indicate that:

- there is a strong correlation between student teachers' levels of mathematics performance and their levels of self-confidence;
- student teachers often hold beliefs about mathematics and learning that constrain their access to rich and powerful ways of learning and teaching;
- many students in teacher education programs believe that calculator use should be avoided in primary mathematics;
- student teachers appear to have had little past experience with activities that might promote number sense or reflection on mathematical processes; and
- many pre-service teachers believe they are insufficiently prepared in terms of mathematics content, pedagogy and pedagogical content knowledge, but believe they are sufficiently prepared in terms of their knowledge of mathematics curriculum.

There is often a dissonance between the beliefs of student teachers and teacher educators, with student teachers believing that good teachers are supportive and enthusiastic, and incorporate relevant, practical, "fun" activities into their teaching, but also believing that teachers are less likely to be empathetic to struggling pupils if they

have a high level of knowledge of mathematics. This needs to be challenged by teacher education programs.

Innovative practices found to be effective in mathematics teacher education included:

- a school-based program in which students took responsibility for teaching a small group of children throughout the year, with findings supporting the view that powerful emotional experiences involving practice and reflection are required if significant and effective change is to occur; and
- the use of an interactive multimedia resource to support student teachers in their study of teaching, resulting in pre-service teachers demonstrating increased observation skills and improved post-practicum discussions.

Further research is needed into effective teacher education, particularly into the ways in which technology can be used to provide student teachers with opportunities to engage in powerful experiences through the structured analysis of carefully selected video and other records of learning and teaching.

Professional development and teacher change

Findings from research on professional development indicate that effective programs:

- provide teachers with the time and appropriate resources to enable them to reflect on their teaching, with teachers seeing a lack of time to adopt new practices as a major impediment to change;
- provide continuing support and encouragement while teachers are exploring possibilities and trialling new strategies in their classrooms;
- involve teachers in school-based and wider networks;
- are of sufficient duration to allow significant changes to habitual beliefs and practices; and
- create opportunities for the exploration of, and reflection on, theory-practice relationships.

Moreover, research in both Australia and overseas has emphasised the importance of professional development being:

- content-focused;
- situated in or near classrooms where teachers work; and
- rooted in the curriculum they teach.

Teachers' use of one-to-one interviews based on developmental frameworks has been found to be a powerful tool for professional growth in programs such as *Count Me In Too* in New South Wales and Victoria's *Early Numeracy Research Project*. By becoming involved in researching pupils' mathematical understandings, teachers' own understandings of how children think mathematically and learn mathematics are enhanced, enabling them to develop teaching approaches and strategies to effectively help children to develop numeracy skills and understandings.

However, there have been few longitudinal studies of teacher change, with only limited knowledge about the longer-term effects of teacher change on students' numeracy outcomes. Research into the extent to which professional development results in sustained change is needed.

Teacher beliefs and knowledge

Australian research in the last ten years on teachers' beliefs is not extensive and appears somewhat fragmented. Much of the research relates to the use of calculators and, to a lesser extent, computers, with research showing that teachers' restricted views on the ways in which calculators can be used is one reason that calculator use has lagged behind support for such use.

International research has emphasised the importance of teachers' knowledge of deep connections between mathematical ideas for high student achievement, yet no Australian research reviewed looked specifically at the mathematical content knowledge of practising teachers — although the recently commenced Australian Research Council (ARC) project *Knowledge for Teaching Primary Mathematics: How Teachers' Pedagogical Content Knowledge Develops and Affects Classroom Practices and Students' Mathematics Achievement*, is investigating the influence of upper primary teachers' mathematics-specific pedagogical knowledge on teaching practice and students' learning outcomes. This project is being undertaken by the Department of Science and Mathematics Education at the University of Melbourne.

However, given that many Australian pre-service teachers feel that their *pedagogical content knowledge* — their knowledge of how to teach mathematics — is inadequate, research appears to be needed into ways in which teachers' pedagogical content knowledge can not only be assessed but also improved. The Japanese "lesson study" model, where groups of teachers in a school undertake self-guided study of particular lessons, discussing in detail the language, materials and activities that can be used to convey underlying mathematical concepts, has attracted considerable international attention as a form of teacher development that addresses teachers' pedagogical content knowledge. This would seem a promising focus for further Australian research.

Students

In projects and publications that focused on students as numeracy learners, *Students at risk* and *Student attitudes* received the most research interest, with less research into students' *Learning styles*, and *Informal learning* and *Gifted* students receiving minimal attention.

Gifted students

A project undertaken by the Australian Mathematics Trust was the only project located that had a focus on mathematically gifted and talented primary-aged children. Otherwise, the few small studies that included mathematically gifted students were limited in scope. Their findings indicated that among children with high mathematical potential, higher-level thinking was promoted through problem solving in small group settings.

The paucity of research on how to develop the full mathematical potential of gifted and talented youngsters needs to be addressed on the grounds of both equity and potential benefit to the community.

Students' informal learning

The number of studies on students' informal learning was relatively small. However, the research indicates that:

- many children begin school with relatively high levels of knowledge, often not acknowledged by teachers;
- many Indigenous children face difficulties due to their culturally different home environments;
- children's mathematical understandings are enhanced when teachers recognise the value of — and establish links with — children's informal, out-of-school mathematics experiences;
- students' informal learning generally appears to be undervalued in the formal curriculum; and
- there is a need to bring into the classroom the more social methods of learning that children experience out of school.

Several current Australian Government funded projects, such as the Australian Council for Educational Research's *Project Good Start: Effective Numeracy Practices in the Year Before and the First Year of School*, are providing information on effective pre-school learning environments, children's numeracy learning at home, and strategies for successful transition from before-school settings to school. There is, however, a need for further research into effective ways in which children's levels of numeracy can be assessed and enhanced at the pre-school level and communicated to teachers on school entry.

Learning styles

In the research reviewed, learning styles arose mainly as one of the factors emerging in studies related to Indigenous students and students with disabilities. Focusing on children's preferred learning styles and providing learning environments that do not conflict with traditional Aboriginal learning styles were found to enhance Indigenous students' learning.

Student attitudes

There has been considerable research into students' attitudes towards and beliefs about mathematics. The findings generally support the notion that positive attitudes and beliefs about a range of issues related to mathematics teaching and learning are associated with enjoyment and motivation, as well as higher achievement.

Specific findings from the research reviewed include:

- a student perception that rules and terminology taught in school have no meaning outside the classroom;
- a belief by students that mental computation is important as mental strategies will be used more outside of school than in school;
- a positive response by students towards the use of technology for mathematics learning, but mixed beliefs about its effects on their learning;
- a positive correlation between students' performance and their beliefs about themselves as learners of mathematics;
- students' self concept being enhanced by positive feedback from teachers on ability and performance;
- negative self-concept having detrimental consequences on attitudes towards mathematics; and

- positive self-identity being identified as a factor contributing to Indigenous students' attachment to school and positive school outcomes.

The Australian Government commissioned study, *Positive Self-identity for Indigenous Students and Its Relationship to School Outcomes* (DETYA, 2000), undertaken by the Queensland University of Technology, recommended that a subsequent, long-term study be carried out to explore the self-identity of Indigenous students. A further Australian Government funded project, *National Indigenous English Literacy and Numeracy Strategy, 2000-2004*, is using the development of greater self-esteem as one of the key strategies to achieve levels of English literacy, numeracy and school attendance for Indigenous students comparable with those of other young Australians.

Students at risk

There has been considerable research and many programs to identify children who are at risk of failing to learn mathematics successfully at an early stage of schooling, with the aim of assisting these children before they become further disadvantaged in their study of mathematics. The identification of students at risk has usually been done through the use of developmental frameworks, such as the assessment framework based on *growth points* developed by Victoria's *Early Numeracy Research Project* or Queensland's *Year 2 Diagnostic Net*.

Findings relating to low achieving children suggest that:

- many of the problems experienced by these students are related to poor language and reading skills;
- low achieving students do not relate contextual word problems to their everyday life;
- such students rely on rules and procedures, with little thought as to whether answers are correct or make sense; and
- low achievers do not possess rich mathematical ideas or have powerful strategies that will enable them to use their mathematical knowledge to improve and enhance their mathematical thinking.

With much of the research focusing on young children, there is a need for more research into the identification of children in middle and upper primary years who are at risk — particularly as research has shown that many children who have not been identified by their primary teachers as potentially at risk in suffer declines in engagement, self-regulation and self-perception in secondary school.

Language factors

Research over the last decade on language factors affecting mathematics learning has been fairly sparse. The research on children from non-English speaking backgrounds has been discussed earlier. However, in the case of Indigenous students, a focus on the development of understanding and use of the language of mathematics in English has been one of the factors shown to be effective in achieving numeracy gains.

Classroom practice

Much of the research reviewed on teaching and learning in the area of numeracy is classroom-based and is discussed elsewhere. This section focuses on *Intervention*,

Grouping, Use of resources, Use of specific teaching strategies, and findings related specifically to Effective teaching of numeracy.

Intervention

Most Australian States and Territories have established early intervention projects and programs focusing on the identification of children with numeracy difficulties. Teachers' use of one-on-one interviews based on developmental frameworks has been found to lead to significant professional growth.

A number of intervention programs have aimed to identify students who have not achieved expected outcomes by a certain stage, in order to give them extra attention. Specific findings related to intervention programs suggest that:

- intervention can have positive effects not only on academic performance but also on general self-concept;
- both individual and group programs can be effective;
- while early-years intervention programs can result in short-term improvements in children's competence and confidence, there may be a need for follow-up attention in later years; and
- student gains are difficult to sustain unless the interventions are articulated back into mainstream classroom pedagogy.

There is a need for research into the longer-term learning outcomes achieved by participants in intervention programs, particularly as the findings of Luke et al. (2003) suggest that, where interventions take the form of withdrawal programs, student gains are difficult to sustain unless the interventions are linked back into mainstream classroom pedagogy. Further research is also needed into numeracy intervention for students at risk as they proceed from primary to secondary school.

Grouping

Substantial differences have been found in ways that effective and ineffective groups work, with research indicating that low achievers may be relatively uninvolved or passive in co-operative small-group work.

Effective small-group learning was generally found to be associated with:

- highly interactive group discussions that involved supporting explanations with evidence;
- genuine engagement with challenging mathematical content and a focus on central ideas;
- co-operative activity in which students use each other as resources, ask questions, and check information; and
- close monitoring by teachers of group work, with on-going assessment of children's higher order thinking and conceptual development.

There has been little research and no clear evidence about the most effective group structures, with researchers in the British Leverhulme project finding the ways that teachers monitor participation and stimulate effective interaction to be more important than the types of groups used.

Further research is needed into the most effective group structures and the roles of teachers during group work. The effectiveness of group work should be compared not only with individual work, but also with models of whole class teaching — such as

that used in Japanese schools — that focus on problem solving and student explanation rather than teacher exposition.

Use of resources

Findings from research over the past decade on concrete teaching materials appear to challenge the assumption that practical teaching aids always help children to develop mathematical understanding. There is now evidence that in some circumstances concrete materials may interfere with rather than facilitate learning, particularly in the early years, and may add to the cognitive demands of numeracy tasks.

Research on the use of technology points to the importance of teachers knowing how to use it to best effect. Research has shown that calculator use can change mathematics curricula and teaching, assist children to develop number sense as well as skills of mental computation, and change teachers' expectations of pupils. There is little evidence, however, of widespread use of calculators as teaching aids, especially in the early years of schooling, with some parents and teachers, and even children, having reservations about their use.

Research into the use of computers has focused on three major areas:

- the use of particular software (both commercially produced and researcher designed) to enhance student learning;
- the use of computer assisted learning, integrated learning systems, and instructional management systems; and, more recently,
- teaching and learning with information and communication technologies.

While some commercially produced software tools have been found to motivate children, enhanced learning outcomes were not necessarily found. However, research on the effects of the development of specific software packages in particular content areas reveals that they can have positive effects on children's conceptual understandings — for example, researcher designed computer games that focused on aspects of decimal understanding were found to be effective in challenging children's misconceptions about decimals.

Research into the use of computer-aided learning found some teachers and principals to be very positive about its use as another resource, rather than as a total approach. There were mixed findings regarding the extent to which its use enhanced learning outcomes, together with an identified need for professional development.

Recent developments in information and communication technologies have led to a focus on on-line teaching, either to supplement regular classroom teaching or to substitute for and complement it in special cases. This is particularly relevant for remote students, students with disabilities, or adult learners studying basic numeracy and literacy content.

There has been little research into textbook use.

Overall, there is a need for research into which aids are most useful in promoting children's numeracy learning, when such aids are most useful, and how to make sure that the use of such aids results in abstraction and generalisation by children. Further research is needed to identify effective pedagogical strategies that supplement and enrich the use of technological tools. The lack of use of calculators, despite the positive research findings, suggests a need for further research into ways in which

classroom teachers can be assisted in incorporating calculators into their everyday teaching and assessment practices.

The use of specific teaching strategies

Specific teaching strategies, which have attracted considerable international research attention, include problem solving and open-ended tasks, with research focusing on cognitive growth, problem contexts, classroom organisation during inquiry-based lessons, and representations that help abstraction and generalised thinking. There has also been growing international attention, especially since the two recent TIMSS video studies, on what is regarded as the common Japanese lesson pattern of basing lessons on a highly structured, in-depth analysis of multiple student solutions to a single problem.

It is clear from the research into problem solving and open-ended tasks that children can construct important mathematical ideas through solving novel problems, and that children at all ability levels are able to become involved in and challenged by the exploration of open-ended questions. Working in groups to solve non-routine problems can provide opportunities for primary children to talk, think, and write mathematically. However, cognitive, social or interpersonal, and external factors have been found to influence the effectiveness of collaboration during small-group problem solving. The use of open-ended tasks with appropriate scoring rubrics has been found to provide important information for teachers regarding different levels of student understanding. Nevertheless, the traditions of school mathematics teaching, learning, and assessment regimes continue to make it a challenge for teachers to adopt open-ended problem-solving approaches in their teaching.

Research findings suggest that listening to children talk about their mathematical experiences provides a rich source of information for teachers about children's learning. Although children's questions in small groups are generally at a lower level than those initiated by the teacher, children in effective groups were found to talk more about mathematical content, propose more ideas, give more explanations with evidence, re-focus discussion more often, and respond to questions more often than children in less effective groups. Unfortunately teachers often saw the enhancement of children's self-esteem as the most important goal of whole-class discussion at the end of lessons, and consequently placed less emphasis on mathematical aspects, such as what constitutes explanation and justification. Similarly it is unfortunate that some students saw conjecturing, criticising, explaining, testing and refining of ideas and procedures primarily as the responsibility of the teacher. Although a high level of support was found among teachers, principals and mathematics educators for the notion of communities of inquiry and purposeful mathematical dialogue, current practice was seen as falling far short of this goal, with a fragmented, outcomes-based curriculum and the use of tasks with very low cognitive demands being seen as obstacles to such practice.

Materials for use by teachers with a focus on realistic problem solving, including the application of mathematics to "real-life" contexts, have been developed in several Australian projects. although relatively little other research in this area was located. Findings from a small number of studies suggest that out of school learning of mathematics is more open-ended than school methods; that students prefer open-ended tasks related to a real life experience or context; and that there is a need to

develop tasks that encourage children to use intuitions built from experience to relate data in graphs to real contexts.

Overall, there has been little empirical, large-scale research on whether the frequent use of open-ended tasks improves outcomes as measured by test performance, whether all children benefit, and which contexts for problems interest children or lead to greater problem-solving success.

Effective teaching of numeracy

There is a lack of agreement in the international research on the characteristics of classroom practice that result in effective teaching of numeracy. This is due to a number of factors including: a lack of agreement on how to measure effective practice, claims that focusing on various aspects of organisational strategies diverts attention from other more relevant factors such as questioning at a high cognitive level, and difficulties in identifying pedagogical variables that result in statistically significant gains in attainment.

Nevertheless, there is considerable convergence in both the international and the Australian research reviewed which suggests that, while effective teachers are not easily characterised and differences in terms of learning outcomes are often small, effective teachers:

- have high expectations that all children, at all levels of primary school, will engage seriously with mathematical ideas;
- emphasise the understanding of mathematical concepts and the connections between these;
- structure purposeful tasks that enable different possibilities, strategies and products to emerge;
- choose tasks that are linked to real situations, engage children and maintain involvement;
- probe and challenge children's thinking and reasoning;
- build on children's mathematical ideas and strategies;
- are confident in their own knowledge of mathematics at the level they are teaching; and
- use assessment as a basis for development of methods and content, and the identification of problems before they affect further progress.

Luke et al. (2003) have called for a focus on renewing mainstream pedagogy in the middle years, identifying a need for research in order to better understand every day classroom practice, as well as the need for a greater emphasis on intellectual demand and student engagement. Many of the possible directions for future research at the primary level identified above resonate with this call.

Moreover, there is growing international interest — especially since the two recent video studies coming out of the *Third International Mathematics and Science Study* (TIMSS) — in research aimed at providing an understanding of every day classroom practice in countries with different cultures and patterns of teaching, as a means of achieving better understanding of one's own practice and looking at ways of extending its boundaries. Most of this research has been carried out at the secondary

level. There is a need for international collaborative research into primary mathematics practice.

Curriculum and Processes

Over a third of the Australian research reviewed related to a range of issues associated with *Curriculum and processes*, with research on the development of particular mathematics concepts clearly dominating other aspects of research in this area.

Concept development

Of the content areas represented in the research into *Concept development*, by far the most research had *Number* as its focus. Of the other areas, much of the research into *Chance and data* was due to one highly prolific research team, while *Measurement*, an aspect of numeracy that is dominant in everyday encounters, received surprisingly little attention.

Algebra

It is well recognised that primary school children's knowledge of arithmetic structure provides the foundational knowledge for their later understanding of algebra. Research however suggests that students fail to abstract from their primary school experiences the mathematical structures that are necessary for them to make a later successful transition from arithmetic to algebra.

Suggestions for ways to smooth this transition include:

- describing and making use of generalisable processes and the structural properties of arithmetic;
- providing classroom activities to address difficulties arising from students' reliance on intuitive language processing, focusing on essential aspects of number knowledge, particularly the notion of equality;
- building on students' capacity to generalise problem situations and to write equations using variables, in order to informally develop the concept of a variable;
- devoting substantial class time for discussion of links between problems, processes used by the children, underpinning concepts, and related ideas; and
- using problem situations in measurement and other non-number areas to develop children's ability to think algebraically.

Chance and data

During the past decade Jane Watson and her colleagues in Tasmania have produced a remarkable body of research into primary school level probability and statistics.

Watson and her colleagues have explored students' understanding of chance in relation to the development of ideas of formal probability and produced a developmental model for its understanding. Based on this model, Watson has suggested the ordering of topics in the curriculum and provided assessment procedures to use in longitudinal evaluation of the implementation of the curriculum.

She has also used innovative pedagogies, such as students viewing video recordings of other students' conflicting responses to chance problems and deciding which response they preferred, finding that the resulting cognitive conflict improved students' understandings.

Many researchers suggest that there is a need to focus on the study of probability in the context of children's everyday experiences, as well as a need for teachers to attend to common misconceptions and intuitive beliefs that tend to inhibit children's development of probability ideas.

In other work related to students' understandings of statistics, Watson and her colleagues used longitudinal data to develop a three-tiered framework for statistical literacy comprising: defining terminology; applying concepts in context; and questioning claims made without proper justification.

Other Australian research in the area of statistics suggests that:

- when dealing with graphs in authentic contexts, students commonly do not
 - appreciate scaling difficulties,
 - relate a graph as relevant in the context of a standard interpretation task, or
 - apply numeracy skills for calculations based on data in graphical representations;
- levels of students' interpretation of data sets are related to their representation skills; and
- students need to be challenged in the classroom using non-standard graphs in order to be better prepared for misleading representations.

Given the importance of graphical literacy in both everyday numeracy and numeracy across the curriculum, children's limited understandings in this area suggest that this should be a priority for future research and teacher professional development.

Measurement

The research reviewed suggests that teachers are unaware of the importance of structuring their teaching of measurement in terms of students' conceptual development in the various sub-strands of *Measurement*. As part of *Count Me Into Measurement* (NSW), a conceptual framework of six levels of increasing complexity has been developed, together with practical activities and sample lesson plans for teaching length, area, volume, and mass. These are based on research findings that indicate the importance of students' knowledge of the unit iteration structure and are aimed at developing students' understanding of measurement concepts and language through the use of informal measurement units before more sophisticated strategies and processes are introduced. Evaluation of the implementation of this program in 38 schools across New South Wales showed highly positive outcomes.

Research on linear measurement indicates that, while most high-ability students appear to have a conceptual understanding of length, the majority of lower-ability students do not appear to acquire important concepts relating to the linear nature of units. Teachers are urged to identify units explicitly when they are teaching measurement, and not rely solely on paper-and-pencil measuring tasks to assess students' understandings of linear measurement.

Among findings from studies investigating children's understanding of area, young children were found to have difficulty visualising the tiling of shapes, suggesting that activities aimed at teaching the "length by width" formula and similar area measurements may not be effective because children do not see the need to cover the area accurately and may need explicit teaching of the row and column structure of rectangular arrays.

Little research was located into the measurement of time.

Overall, Australian research into measurement over the past decade highlights the importance of devoting sufficient time to the development of underlying concepts before moving to paper-and-pencil activities and formulae.

Given the importance of measurement in everyday numeracy and the need to estimate and carry out mental computations related to measurement, there appears to be a need for research into computational aspects of measurement and ways in which connections between measurement and our decimal system can be used to support students' acquisition of numeracy skills.

Number

The prominence of number in primary mathematics is reflected in the quantity of research projects identified in this area, with the two major areas for research into *Number* in the past decade having been *Computation* and *Number sense*.

A substantial amount of Australian research into children's early number learning has informed the development of frameworks for teachers to use in assessing young children's stages of conceptual development. These frameworks have underpinned research and professional development programs, such as *Count Me In Too* and Victoria's *Early Numeracy Research Project*, which aim to enhance children's numeracy outcomes by enabling teachers to engage with their students' mathematical learning in order to create challenging activities for all children. Teachers working with "at risk" students in intervention programs have been urged to share their knowledge with classroom teachers in order that they too can use this knowledge to develop appropriate classroom activities for these students.

Studies looking at children's learning of number in the early years of school suggest that many children are seriously "under-challenged" and emphasise the need to give problem solving, as well as abstract mathematics, a more central role. Many researchers have called for a more holistic approach to the teaching of number, with an emphasis on connections rather than compartmentalised knowledge.

The role of concrete materials in the development of place value concepts, including the underlying concepts involved in decimal notation, has been questioned, with teachers being urged to make explicit the connections between the materials and place value, and to have a repertoire of activities to develop and reinforce place value concepts at a more abstract level. In terms of the teaching of decimals, student improvement has been found to depend on teachers' knowledge of the underlying concepts, the use of a clear model and careful bridging from visualisation to numerical forms. A disturbing finding is that, for some children at least, certain misconceptions relating to decimals appear to be learned from school instruction.

The use of various models for the development of both mental and written addition and subtraction has been the subject of considerable research. Tens frame have

been found useful for developing children's thinking in tens, with children being able to build up basic facts without resorting to primitive counting strategies. A major new model for addition and subtraction to 100, the *empty number line*, has been developed in the Netherlands as part of *Realistic Mathematics Education*. This model, which has been adopted in many places, including Australia as part of *Count Me In Too*, is seen to provide a natural and transparent means to model children's informal strategies and has been found to be a powerful model for instruction in the Netherlands.

Researchers investigating children's conceptual development across a wide range of aspects of number frequently highlight the need for teachers to identify, value, and develop children's spontaneous, informal computational strategies. Studies of children's mental computation show that many competent children have acquired a range of efficient strategies almost "in spite of" what happens in the classroom. These efficient strategies are flexible, taking into account the numbers involved, unlike those that merely mirror standard written algorithms.

The place of standard written algorithms in the mathematics curriculum and their role in children's development of number sense continues to be the subject of debate. A wide range of studies into children's arithmetic strongly suggest the need to place more emphasis on children's understandings of fundamental concepts before the teaching of rules and procedures, as well as the need to coordinate new symbolic knowledge with children's existing informal knowledge and their real-life experiences.

Research indicates that number sense and mental computation are closely linked.. This suggests that mental computation should be given greater prominence in school curricula at the expense of the teaching of standard written algorithms, and that more emphasis should be placed on the assessment of mental computation. Among the outcomes of a continuing international collaborative program of research into number sense (Number Sense Research Group: Reys, Reys, McIntosh, et al.) mental computation and computational estimation has been the development of a framework for the analysis of number sense and the development of paper-and-pencil tests for number sense and mental computation.

Research into the role of calculators in children's early number learning suggests that they have the potential to develop children's conceptual understanding and mental computation strategies before the formal teaching of algorithms, with their presence providing a learning environment to promote number sense. While *appropriate* choice of computational methods is an indicator of number sense, a significant factor in students' computational choices appears to be their belief that mental and written methods are more highly valued than using the calculator. Rather than categorising computation as mental, calculator and written (in the sense of standard written algorithms), it has been suggested that "written" be replaced by "recording", which then includes children's informal jottings during the calculation as well. The Tasmanian project *Developing Computation* funded through the Australian Government's *Strategic Numeracy Research and Development Project*, is supporting the development of informal written methods in years 2 to 4, while also investigating the effects of such a program on students' number sense and computational ability.

In summary, findings from the extensive body of research on *Number* suggest that:

- many children are seriously “under-challenged” in their learning of number in the early years of schooling;
- explicit connections need to be made between concrete materials and the concepts being developed, with further activities needed to develop and reinforce concepts at a more abstract level;
- teachers’ knowledge of the underlying concepts, the use of a clear model, and careful bridging from visualisation to numerical forms are important factors in the effective teaching of decimals;
- more emphasis is needed on children’s understandings of fundamental concepts before the teaching of rules and procedures;
- new symbolic knowledge needs to be coordinated with children’s existing informal knowledge and their real-life experiences; and
- the use of calculators as teaching aids can enhance children’s conceptual understanding and mental computation before the formal teaching of algorithms.

As a result of their findings, researchers have called for:

- more emphasis on problem solving and abstract mathematics;
- a more holistic approach to the teaching of number, with an emphasis on connections rather than compartmentalised knowledge;
- more focus on children’s spontaneous, informal computational strategies;
- greater prominence in school curricula for mental computation at the expense of standard written algorithms; and
- more emphasis on the assessment of mental computation.

Further research is needed into:

- which aspects of schooling and which pedagogical approaches are most effective in developing children’s number sense; and
- the development and use of new models and materials for instruction in mental and written computation.

Space

A developmental framework for two of the sub-strands of *Space*, based on extensive collaborative research into visual imagery and spatial thinking, has been constructed as part of the *Count Me Into Space* project. This has been accompanied by suggested lessons and sets of assessment tasks, which have been used for professional development for early years teachers. Detailed evaluations showed project students performing significantly better than those from comparable schools, while teachers reported many positive aspects, such as feeling that they had extended their mathematical and pedagogical content knowledge. An *angles* unit, with a corresponding teaching package, has also been developed as part of *Count Me Into Space*.

Research into children’s use of diagrams — which play an important role in the communication of geometric ideas — shows that children may experience serious difficulties in interpreting diagrams and that this may be a constraint to effective communication in geometry. Other research into children’s spatial ideas and their ability to express them suggests that young children are more developed in spatial

concepts than they can verbalise, and that their perceptions of shapes and their approaches to tasks are strongly influenced by language and experience.

Spatial encounters occur in a range of diverse everyday environments, suggesting that teachers need to be made more aware of key concepts and experiences that can be drawn out of everyday environments. While the importance of building on children's experience needs to be emphasised, a study of adults' concepts of space, time and money in a remote rural Aboriginal community highlights the radically different views and socio-cultural experiences of people in "non-typical" environments.

In summary, findings from the research on *Space* indicate that:

- positive results in terms of student learning outcomes and teacher satisfaction resulted from the use of a developmental framework and lessons in the *Count Me Into Space* project;
- children's interpretations of diagrams can constrain their effective communication in geometry;
- language and experience strongly influence young children's perceptions of shapes and approaches to tasks, with spatial concepts often being more developed than children can verbalise; and
- teachers need to be made more aware of key concepts and experiences that can be drawn out of everyday environments in the early years of schooling.

Given the importance of spatial awareness in terms of everyday numeracy, and the success of the *Count Me Into Space* project, further research is needed into the types of tasks and teacher actions that can enhance students' learning in the area of space.

Developmental frameworks

The construction of developmental frameworks, based on extensive research, particularly in the early years, has been a major feature of recent Australian research. Wright's *Learning Framework in Number*, for example, which was based on research by Steffe and others in the United States, defined levels of competence in children's identification of numerals, forward and backward counting sequences, understanding of place value, and addition and subtraction problems. This learning framework has underpinned much of the Australian research into children's early learning of counting and arithmetic, including the New South Wales Department of Education and Training's intervention program in which "at risk" year one students were withdrawn for one-to-one long-term teaching programs — the beginnings of the *Count Me In* program. Similar, though less extensively research-based, frameworks have also been developed in New South Wales in the areas of *Measurement* and *Space*.

Another example of a developmental framework has been the framework of *growth points* of early numeracy learning developed in Victoria by the *Early Numeracy Research Project*. The framework was used to develop a task-based, one-to-one interview schedule that was used with children as part of a multi-level professional development program. Use of this interview by teachers has been seen as a powerful professional development tool to engage teachers in investigating the mathematical knowledge of their students and hence enabling them to better plan appropriate

sequences of learning tasks. This research has also led to the development of a new framework including counting, place value, strategies for addition and subtraction, strategies for multiplication and division, time, length, mass, properties of shape, and visualisation and orientation, as part of the Victorian *Early Years Numeracy Program, P-4*, which is being used as a basis for materials production, monitoring and assessment of students' development, and professional development.

The Western Australian *First Steps in Mathematics* project developed detailed diagnostic maps and associated curriculum materials, together with a structured professional development program aimed at enabling teachers to link the number, measurement and space strands in the student outcome statements with curriculum content in each phase of schooling.

In summary, Australian research into stages of children's development, particularly in number, has resulted in the construction of research-based learning frameworks in several States.

These developmental frameworks have been found to be useful for:

- mapping students' progress;
- developing appropriate curricula; and
- linking teacher professional development to key mathematical concepts and their development.

Moreover, their use in projects such as *Count Me In Too* and the *Early Numeracy Research Project* has been found to result in improved learning outcomes for children and increased satisfaction for teachers.

Further research is needed at both the pre-school and primary-secondary transition levels in order to provide a common language that enables clear communication of children's capabilities and needs between pre-school staff, primary and secondary teachers, and parents.

Mathematical thinking

The critical role of mathematical thinking in the application of mathematical knowledge and skills to real-life situations gives it a central role in numeracy.

Of the research reviewed under *Mathematical thinking*, the themes of *Children's thinking strategies* and *Children's problem solving* attracted considerable attention.

Children's problem solving

Studies of children's deductive reasoning have shown that even children classified as low achievers in mathematics can reason logically when solving novel problems and can develop sophisticated procedures, suggesting that the mathematics curriculum needs to be broadened to include opportunities for all children to attempt novel problems and have more control of their learning. Research into children's problem posing indicates a lack of connection between children's informal, intuitive knowledge and "school maths", suggesting that explicit attention needs to be paid to encouraging children to make these connections and recognise mathematical situations in everyday life.

Teaching a range of problem-solving strategies and metacognitive practices, such as planning an approach, and monitoring one's progress, has been shown to increase

problem solving effectiveness. Research has also shown perseverance to be a contributing factor in successful problem solving.

Another factor influencing successful problem solving is the effective use of diagrams, which suggests that numeracy and literacy needs to be expanded to include facility with various forms of visual representation, including diagrams.

In an attempt to provide information for teachers about the correctness of children's problem solutions and the problem solving processes the children are using to arrive at their solutions, a number of Australian assessments of problem solving have been developed. These also provide teachers with information on which to base their subsequent teaching.

In summary, findings from the research on *Children's problem solving* indicate that:

- children classified as low achievers in mathematics can reason logically in solving novel problems and develop sophisticated procedures;
- there is a lack of connection between children's informal, intuitive knowledge and "school maths";
- children need to be encouraged to recognise mathematical situations in everyday life;
- successful problem solvers monitor and manage their progress in selecting and implementing appropriate strategies;
- the use of metacognitive models can enhance problem solving performance;
- the effective use of diagrams affects successful problem solving; and
- teachers need to use a range of assessment techniques to obtain a complete picture of children's problem solving.

Children's thinking strategies

Research into children's thinking strategies has been closely linked to research into problem solving.

Findings from the research on *Children's thinking strategies* indicate that:

- children's ability to monitor their actions, detect and correct errors, and recognise problem completion play a crucial role in successful problem solving;
- although children reason by analogy in everyday life, they appear to require guidance to apply this to more formal problem solving; and
- while students increasingly use formal methods as they move to higher year levels, teachers play a significant role in student success in formal responses.

In an attempt to bridge the research-practice gap, a research-based pedagogical framework to promote mathematical thinking and understanding in mathematics classrooms has been developed. The need for a thinking curriculum to be accompanied by assessment practices that support thinking and sense making has been stressed, as well as the value of providing teachers with "descriptive images" of practices to promote mathematical thinking.

The language of mathematics

The abstract, compressed nature of mathematical language, with its focus on conventions and written symbols, and its use of familiar words in unfamiliar ways, presents barriers to student learning of mathematics.

Research suggests that in order to enable children to link their own mathematical ideas with formal mathematical language, teachers should:

- be aware of the discontinuities between children's out of school use of language and mathematical language;
- allow children to use their own words to explore and express their mathematical ideas and thinking; and
- systematically plan classroom activities that develop young children's understanding of concepts.

Visualisation

The visualisation of mathematical concepts has long been recognised as important for mathematical thinking. Australian research has looked at the role of visualisation in the development of number concepts, spatial understanding, and problem solving.

An important area of research in both Australia and internationally has been the link between mental images and number sense of young children. Investigation of the role of equipment, drawings and activities, to enhance children's thinking strategies and promote number sense, suggests that the active processing of images plays an important part in children's development of number concepts.

Both visual and non-visual reasoning have been found to play an important role in problem solving, with students often using visual processing in the initial stages of problem solving, and then moving to more analytic, non-visual strategies. While drawing a diagram is recognised as a powerful problem solving strategy, students have been found to experience a range of difficulties in generating effective diagrams in problem solving.

In summary, the research into *Mathematical thinking* has been dominated by research into the two overlapping areas of *Children's thinking strategies* and *Children's problem solving* and has focused largely on cognitive aspects of problem solving. Unlike other countries, such as the United States and Japan, there has been very little focus in Australia on systematically developing children's algebraic reasoning, processes of generalisation and proof, or their ability to represent and interpret mathematical expressions signifying the relationships between quantities. This is a rich strand of the mathematics curriculum in some other countries (e.g., Japan) from middle primary school mathematics onwards. Such a focus underpins students' abilities to use information technology tools appropriately as well as to think at higher levels and to solve challenging problems.

Further Australian research is needed into the development of conceptually focused, robust tasks and new teaching approaches that can be used to support the development of sophisticated mathematical thinking.

Using mathematics

In a mapping and review of research into numeracy it has been disappointing to find almost no research into children's use of mathematics in everyday situations and other non-mathematical contexts. While there may be accounts in professional journals of children applying mathematical concepts and skills in a variety of situations, only two ARC funded projects relating to mathematical modelling and a

project on the application of mathematical ideas to discussions of stories in class were found.

Given that using mathematics is where children's numeracy can be put into practice, this is clearly an area where further research is needed.

Assessment

Research projects and publications on primary numeracy that used assessment of achievement as a major criterion for effective teaching are discussed elsewhere, with only those studies whose major focus was assessment being included here.

Assessment programs

In the international sphere, the most significant assessment program in the past ten years has been the *Third International Mathematics and Science Study* (TIMSS).

Findings relating to Australian children's performance in TIMSS showed that:

- children in only six countries outperformed Australian children overall;
- Australian children were the highest achieving English-speaking children;
- at the upper grade level, Australian children were equal top achievers in the content area of space;
- Australian primary children performed relatively poorly in the area of whole numbers;
- there were no statistically significant differences in mathematics achievement between boys and girls;
- Indigenous children scored significantly lower than non-Indigenous children; and
- children who spoke English at home achieved higher scores than those whose home language was not English.

Nationally, numeracy benchmarks have been agreed upon and common assessment items are being included in statewide testing programs. National benchmarks have been developed for use in reporting minimum acceptable standards of literacy and numeracy achievement, in support of the national literacy and numeracy goal. While comparisons between the draft Year 3, 5 and 7 benchmarks, international curriculum documents and achievement data from TIMSS suggest that statements of expectation are set at higher levels in some other countries, the Australian benchmarks are agreed minimum standards, which is not necessarily the case elsewhere. The data provides nationally comparable information on student achievement, which will facilitate the monitoring of trends over time and in relation to the national goal (see p.2). Importantly, the data produced includes information on the performance of sub-groups of students.

All States and Territories now have large-scale assessment programs in place. These tests monitor detailed student performance within education systems in terms of the relevant curriculum. National benchmark data is also obtained from these tests.

There has been little research into the advantages, limitations and effects of adopting alternative forms of system-wide assessment on the mathematical achievements of children, forms of assessment that take into consideration differences in children's

learning styles, and the difficulties and limitations that assessment places on those with language, physical and cognitive disabilities.

Assessment techniques

Over the past decade, research on *Assessment techniques* has been concentrated in three main areas: techniques for assessing particular aspects of mathematical learning — for example, problem solving or statistical understanding; developmental assessment — for example, the construction of developmental frameworks in early number learning or mental computation; and the use of innovative assessment styles — particularly so-called *Rich Assessment Tasks*.

While much of the research into the first two of these areas has been discussed elsewhere, it is worth noting here that there has been a growing interest in and use of Item Response Theory (IRT) and Rasch measurement to devise numeracy scales to enable the numeracy development of individuals or groups to be monitored, as well as an increasing interest in the use of developmental assessment frameworks as a catalyst for driving changes in instructional decision making.

In keeping with criticisms of research studies and classroom practices that use traditional pencil-and-paper tests as the sole measures of numeracy achievement, there has been a growing interest in and use of *Rich Assessment Tasks*. These tasks emphasise problem solving and reasoning, create links between different mathematical topics, require students to apply a range of conceptual and procedural knowledge, and often incorporate simulations of real life situations. Projects such as Victoria's *Middle Years Numeracy Research Project* have incorporated the use of rich assessment tasks into the design and development of their *Student Numeracy Profile*, with results suggesting that their use has the potential to provide useful insights into instructional strategies for middle years' students. Rich assessment tasks have also been a key component of recent and current projects in several States and Territories, including the Northern Territory, where their use with middle years Indigenous students is being investigated.

Findings from many of the current Australian Government *Strategic Numeracy Research and Development Projects* will make a significant contribution to research in these areas.

Diagnostic and school entry assessment

Much of the focus for *School entry assessment* has been on the identification of children "at risk" and those considered to have need of numeracy intervention. Various State and Territory educational systems have implemented school entry assessment projects and programs with some extending beyond the first year of schooling.

However, further research appears to be needed into the development of appropriate tools for profiling pre-school children's mathematical development in order to smooth the transition between pre-school and school.

Similarly, while the transition from primary to secondary school has been identified as a particularly critical period of children's schooling, and several numeracy-based projects and research studies have included data gathered from year levels that straddle this transition, assessment issues were not the major focus of any of these

projects. In fact, overall, it appears that diagnostic assessment, other than in the early years of schooling, has not received much systematic research attention.

Conclusion

While there is an impressive array of Australian research into primary numeracy, there are still many areas where further research and development are needed. In order to achieve the goals agreed by State, Territory, and Australian Government Ministers for Education, that all students should attain the skills to be numerate, it is important that numeracy research remains a high priority for Australian research.

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